Enrollment No: ____________________ **Exam Seat No:** ____________________

C. U. SHAH UNIVERSITY

Winter Examination-2019

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT1 **Branch: B.Tech (All)**

Semester : 1 **Date : 16/11/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

1. (1) Use of Programmable calculator & any other electronic instrument is prohibited.
2. (2) Instructions written on main answer book are strictly to be obeyed.
3. (3) Draw neat diagrams and figures (if necessary) at right places.
4. (4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $1,\ \alpha_1,\ \alpha_2,\ .....,\ \alpha_{n-1}$ are the n roots of unity, then $(1+\alpha_1)(1+\alpha_2).....(1+\alpha_{n-1})$ is equal to

(A) $n-1$ (B) n (C) – 1 (D) none of these

b) The polar form of the complex number $\frac{1+i}{1-i}$ is

(A) $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$ (B) $\sin\frac{\pi}{2}+i\cos\frac{\pi}{2}$ (C) $\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$

(D) $\sin\frac{\pi}{4}+i\cos\frac{\pi}{4}$

c) If $f(x)=\frac{e^x-e^{-x}}{2}$ is continuous at $x=0$, then the value of $f(0)$ must be

(A) 0 (B) 1 (C) 2 (D) 3

d) $\lim\limits_{x\to\infty}\frac{x^n}{e^x}=$ _____

(A) 0 (B) 1 (C) 2 (D) none of these

e) The infinite series $1+r+r^2+.....+r^{n-1}$ is convergent if

(A) $|r|<1$ (B) $|r|>1$ (C) $r=1$ (D) $r<-1$

f) The series $\sum_{n=1}^{\infty}\frac{1}{(\log n)^n}$ is

(A) oscillatory (B) divergent (C) convergent (D) none of these

g) If the power of *x* and *y* are even, then the curve is symmetrical about

(A) X-axis (B) Y-axis (C) about both X and Y axes

(D) none of these

h) The asymptotes obtained by equating coefficients of highest degree terms in *x* to zero are called asymptotes

(A) parallel to X-axis (B) parallel to Y-axis (C) oblique
(D) none of these

i) If $y = x - \frac{x^2}{2!} + \frac{x^3}{3!} - .....\infty$ then x equal to

(A) $y - \frac{y^2}{2} + \frac{y^3}{3} - .....$ (B) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + ...$ (C) $y + \frac{y^2}{2} + \frac{y^3}{3} + ...$
(D) none of these

j) If $y = \cos^{-1} x$, then x equal to

(A) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - .....$ (B) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - .....$ (C) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!}$
(D) none of these

k) If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{\partial f / \partial x}{\partial f / \partial y}$ (B) $\frac{\partial f / \partial y}{\partial f / \partial x}$ (C) $-\frac{\partial f / \partial y}{\partial f / \partial x}$ (D) $-\frac{\partial f / \partial x}{\partial f / \partial y}$

l) If $u = f\left(\frac{x}{y}\right)$ then

(A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ (B) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ (C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$

(D) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

m) If $x = r\cos\theta,\ y = r\sin\theta,$ then $\frac{\partial(r,\theta)}{\partial(x,y)}$ is equal to

(A) 1 (B) r (C) $1/r$ (D) 0

n) Conditions for $f(x, y)$ to be maximum are

(A) $f_x = 0 = f_y,\ rt < s^2,\ r < 0$ (B) $f_x = 0 = f_y,\ rt > s^2,\ r < 0$
(C) $f_x = 0 = f_y,\ rt > s^2,\ r > 0$ (D) $f_x = 0 = f_y,\ rt = s^2,\ r > 0$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Prove that $(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2(a^2+b^2)^{\frac{m}{2n}} \cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$. **(5)**

b) Evaluate: $\lim_{x \to 0}\left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{\frac{1}{x}}$ **(5)**

c) Evaluate: $\lim_{x \to 1}\left(\frac{x}{x-1} - \frac{1}{\log x}\right)$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Expand $\sin^5\theta\cos^2\theta$ in a series of sines of multiples of θ. **(5)**

b) Evaluate: $\lim_{x \to 0}\frac{a}{x^2}\left[\frac{\sin kx}{\sin lx} - \frac{k}{l}\right]$ **(5)**

	c)	If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$ then prove that $\lim\limits_{n\to\infty} x_1x_2x_3\ldots x_n = -1$.	**(4)**
Q-4		**Attempt all questions**	**(14)**
	a)	Prove that $\cos^{-1}\left[\tanh\left(\log x\right)\right] = \pi - 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\ldots\right)$.	**(5)**
	b)	Expand $f(x) = \frac{e^x}{e^x+1}$ in powers of x up to x^3 by Maclaurin's series.	**(5)**
	c)	Test the convergence of the series $\sum\limits_{n=2}^{\infty}\frac{1}{n\left(\log n\right)^2}$.	**(4)**
Q-5		**Attempt all questions**	**(14)**
	a)	Test the convergence of the series $\frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}-1} + \ldots\ldots$	**(5)**
	b)	Examine the series $\sum\limits_{n=1}^{\infty}\frac{x^n}{n^p}$ for convergence using root test.	**(5)**
	c)	Expand $\log x$ in powers of $\left(x-2\right)$.	**(4)**
Q-6		**Attempt all questions**	**(14)**
	a)	If $u = \sec^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\cot u\left(\cot^2 u + 2\right)$.	**(5)**
	b)	If $u = \log\left(x^3+y^3+z^3-3xyz\right)$ then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{\left(x+y+z\right)^2}$.	**(5)**
	c)	Find the asymptotes of the curve $y^3 - x^2\left(6-x\right) = 0$.	**(4)**
Q-7		**Attempt all questions**	**(14)**
	a)	Trace the curve $xy^2 = 4a^2\left(2a-x\right)$.	**(5)**
	b)	If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$, evaluate $J = \left(\frac{x,y}{u,v}\right)$ and $J' = \left(\frac{u,v}{x,y}\right)$ and hence verify that $JJ' = 1$.	**(5)**
	c)	Find the approximate value of $\sqrt{27}\sqrt[3]{1021}$ using partial differentiation.	**(4)**
Q-8		**Attempt all questions**	**(14)**
	a)	Trace the curve $r^2 = a^2\cos 2\theta$.	**(5)**
	b)	Find the maximum and minimum values of $2\left(x^2-y^2\right) - x^4 + y^4$.	**(5)**
	c)	The period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Find the maximum error in T due to possible errors up to 1% in l and 2.5% in g.	**(4)**